On the Reasons of Hyperbolic Growth in the Biological and Human World Systems

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Abstract
Macroevolution of the biological and human world systems in the aspect of time-dependence of their sizes is studied. These systems are considered as civilizations that, by definition, have memory and produce knowledge (vital information). Sizes of three types of memory – genetic, neural, and external – are estimated. Dominating one of them leads to the development of an appropriate type of civilization. The rise and development of genetic memory was accompanied with the formation of biota (which can be tractable as a biological civilization) and hyperbolic growth of its biodiversity. The prevailing development of neural memory in one of the taxa of biota led to the rise of human civilization and hyperbolic growth of its population. The development of external memory will probably lead to the extraction of a taxon (seemingly, a group of countries) from the human world community, hyperbolic growth of its memory and fund of knowledge (but without pronounced growth of population).

Key words: macroevolution; human population; biodiversity; genetic, neural and external memory types; knowledge accumulation; hyperbolic growth.

1. INTRODUCTION

Demographic data show that, at least several tens of thousands of years, almost up to the end of XX century, human population growth followed a hyperbolic law (Foerster, Mora, and Amiot, 1960; Hoerner, 1975). Moreover, it appears that a similar hyperbolic law is also inherent in the diversity of different taxa in the biological world system, indicated by the growth of the number of families and genera in the marine and continental biota (sepa-
rately and as a whole) during the Phanerozoic (Markov and Korotaev, 2007, 2008, 2009; Grinin, Markov, and Korotaev, 2009). The similarity of growth laws suggests that in both cases there is a universal mechanism bringing such different systems to the same regime of growth. In the cited works on biodiversity, the hyperbolic growth law is associated with an increase in lifetime of taxa (families or genera; data on species are not reliable). However, lifetime itself depends on the fit of taxa to ambient conditions and therefore is determined by the content of valuable information accumulated in genomes. Finally, it turns out that there are informational reasons for inducing the biodiversity growth. Similar reasons are responsible for the hyperbolic growth of human population; this growth is a result of accumulating valuable information in genetic memory and, in a much more rapidly way, in neural one (Dolgonosov and Naidenov, 2006; Dolgonosov, 2009). Thus, similar informational mechanisms regulate sizes of both world systems, namely, human population and the diversity of taxa in biota.

From the informational viewpoint, civilization represents a system having memory and producing knowledge. This understanding of civilization spreads the action of this term on systems of any (not only human) origin, because the presence of memory equipped with a processor for extracting valuable information from incoming signals is a feature of not only the human but every biological species. In this connection, the question rises how memory type influences the type of civilization.

Along with genetic and neural memory, there is external memory. The first two types of memory are an internal property of biological units constituting civilization. External memory is inherent in the human civilization, where this memory type is realized in the form of different external carriers of information: physical samples, books, films, computer carriers, etc. Genetic memory dominates in the biota providing accumulation of valuable information and its inheritance. Thus, biota demonstrates the above mentioned attributes of civilization that allows it to be named the biological one. Human civilization, grown from it, had used advantages of neural memory (primarily, speed of processing information), whose size on a definite stage of phylogenesis achieved the size of genetic memory and then surpassed it providing an accelerated development of this phyletic branch. Further evolution of humanity had led to development of external memory, which became dominant in our time due to the fast perfection of computer carriers.

Let us consider the macroevolution of civilization under domination of a definite memory type and what happens when the dominant changes. An important role pertains here to the compression of information in its processing into knowledge.
2. COMPRESSION OF INFORMATION

Each type of memory is provided with a processor, which transforms unconditional (primary) information $R$, perceived through signals from the outer world, into conditional (useful, valuable, vital) information $q$ that represents knowledge (Fig. 1). As measures of the quantities $R$ and $q$, we can take the memory sizes needed for storing of corresponding information. Primary, unconditional information requires too large memory size to store it completely and, moreover, cannot be used directly, without preliminary systematization. Primary information can be processed into knowledge only if there is a model of data structure and an algorithm of data compression (Salomon, 2007). The model and the algorithm are necessary for solving tasks of image recognition and for choosing behavior. The most significant compression – logarithmic – is a result of selection in the evolutionary process that allows extracting vital information from all the data.

![Fig. 1. Schematic of knowledge production.](image)

Logarithmic compression occurs at a transition from a microscopic description considering each of the unimaginable number of microstates of the system to a macroscopic description using a finite, mostly not large, set of macro-variables. In statistical physics, this corresponds to the concept of statistical ensemble of microstates at fixed values of macro-variables such as temperature, pressure, volume, particle numbers, etc. In such a transition, compression of information is given by the entropy $S(X) = \ln W(X)$, where $X$ is a set of essential macro-variables, $W$ is the statistical weight, or the number of micro-
states at a fixed $X$ (Zubarev, Morozov, and Röpke, 1996). In our terms, $W$ represents the primary information, and $S$, valuable one – knowledge ($R$ and $q$, respectively).

Thus, the logarithmic law of information compression is a result of extraction of valuable information and can be written in the form $q = q_c \ln(R / R_0)$, where $q_c$ is a constant depending, in general, on memory type; $R_0$ is the minimum memory size at $q \to 0$. Notice that, if there is a dominant memory type, the knowledge size $q$ can be measured in the units of $q_c$ and the primary information $R$, in the units of $R_0$, i.e. formally we can set $q_c = 1$ and $R_0 = 1$, and then the information compression law takes the form $q = \ln R$.

This law can be interpreted as follows: at a knowledge level of $q$ civilization is capable to recognize primary information from the outer world in the amount of $R = e^q$, i.e. figuratively speaking, the scope of civilization is exponentially expanded with knowledge.

As stated above, the common memory of civilization contains primary information and knowledge. The former is accumulated partially so far as the available memory size allows. Because of $R >> q$, a prevailing part of memory of any type is filled with raw, partially processed information.

3. MEMORY SIZES

In (Dolgonosov and Naidenov, 2006; Dolgonosov, 2009) an attempt to estimate non-redundant memory of different types has been made. Here we consider total available memory of each type.

In the case of genetic memory, primary information is processed into genetic code under natural selection. Of all information accumulated in genome, the fraction of vitally important (coding and regulating) part depends on genome size. For example, in a comparatively small genome of bacteria (nearly 1 Mbase) it takes about 50%, and in an advanced human genome (3200 Mbase), about 5% (Lesk, 2002; Yankovsky, 2009). The role of the rest part of genome has not been clear yet; it is only known that this part is a powerful internal source of mutations (Lander et al., 2001). Possibly, it is the very partially treated information that is being processed into knowledge (i.e. into vital information). Now, the total size of genetic memory of humanity with the current population of 6.7 billion is $2 \cdot 10^{19}$ base pairs.
In the case of neural memory, primary information is processed in the course of intellectual activity that provides elaborating rules of recognition and behavior, they being fixed in the memory under permanent usage. The size of brain memory occupied with these rules is likely small compared to the size occupied with raw, partially processed information, though it is hard to estimate these sizes quantitatively. It is known (Bekhtereva et al., 1982; Blum et al., 1988; Haken, 2001; Ndahabaliye, 2002) that human brain consists of $10^{10} - 10^{11}$ neurons. In interval estimations by orders of magnitude, the middle of interval should be found using the logarithmic average, which in this case is $3 \cdot 10^{10}$ neurons. Hence, taking into account the current human population, we find the total human neural memory of $2 \cdot 10^{20}$ neurons. Thus, comparing the numbers of elements, we can conclude that neural memory has already surpassed genetic one. The transition from the domination of genetic memory to that of neural one occurred probably on early stage of anthropogenesis dated 5 – 7 million years ago, when brain volume of hominids was several times less than that of the contemporary human. Genome size has little changed for this period. Regarding that it was $\sim 10^9$ b.p. and population size was $\sim 10^5$ individuals, we can estimate the total genetic memory of $\sim 10^{14}$ b.p. Most likely, there was about the same number of elements in the total neural memory in that time (Fig. 2).

![Fig. 2. Growth of the total memory size during anthropogenesis and historic time. Intervals of domination of genetic (G), neural (N), and external (E) memory are shown. Values on the plot indicate the number of memory elements at the moments of transitions.](image)
At present, of different kinds of the external memory, computer carriers dominate. As Gubailovsky (2005) reported referring to the researchers from the University of Berkley, in 2002 humanity produced $18 \times 10^{18}$ bytes $= 1.4 \times 10^{20}$ bits of information, a prevailing part of which was written on magnetic carriers. Taking into account the storage for all previous years up to date, we can roughly estimate a current size of external memory of $10^{21} - 10^{22}$ bits. Thus, at the moment, external memory has already surpassed neural one by the number of elements. The transition happened at the turn of XX–XXI centuries (Fig. 2) and was accompanied with qualitative changes: if earlier the total memory size of civilization rose proportionally to its population (because the memory was internal), after this event external memory had started dominating and growth of population became unnecessary for an expansion of memory, because this expansion could be provided solely at the expense of external carriers.

4. INFORMATIONAL ORIGINS OF THE HYPERBOLIC LAW

All the long-term evolution preceding the informational epoch is characterized by domination of internal memory, whose size $R$ is the sum of the memory size $m$ of each of the $N$ individuals yielding $R = mN$. Assuming that the minimum memory $R = 1$ (in relative units) inherent in the incipient civilization corresponds to an initial population size $N_0$, we obtain $m = 1/N_0$ and hence $R = N/N_0$. Appealing to $R = e^q$, we obtain $N = N_0e^q$. It means that, in the evolution, both the size of recognizable primary information and the population size of civilization grow exponentially with accumulating knowledge.

Total rate of knowledge production is determined by the sum of individual contributions and therefore is proportional to population size: $dq/dt = wN$, where $w$ is the average knowledge production rate per individual. This yields the equation of knowledge production $dq/dt = wN_0e^q$, which can be written in such a short form $\dot{q} = q^2$ without parameters (point denotes time derivative). It is seen that the process develops in the blow-up mode. It is accompanied with an accelerated growth of population. Actually, from the equation of knowledge production and using the relationship $q = \ln(N/N_0)$ it is easy to derive the equation of population dynamics $dN/dt = wN^2$. Its solution gives the well-known hyperbolic law of population growth $N = N_0/(1 - wN_0t) = w^{-1}(t_1 - t)^{-1}$, where
\[ t_i = 1/(wN_o) \] is the moment of singularity. The expressions obtained show that the higher the individual production \( w \), the faster population grows and the earlier moment of singularity is reached.

Thus, it appears that the hyperbolic law found initially in an empirical way for global human population (Foerster, Mora, Amiot, 1960) and recently for biodiversity (Markov and Korotaev, 2009; Grinin, Markov, and Korotaev, 2009) has a purely informational origin. The above theoretical reasoning suggests that this law must be satisfied for any civilization, in which individual properties such as memory size \( m \) and knowledge production rate \( w \) are conserved and information is accumulated in the internal memory channels. Certainly, the conservation of parameters \( m \) and \( w \) should consider as an idealization that, strictly speaking, does not answer the reality. It is known that for the last 5–7 million years of anthropogenesis brain volume has roughly four times increased (Markov, 2004). It should be expected that some increase in the information processing rate has also occurred being caused by raising the connectivity of neural network in brain due to intensification of intellectual activity. However, in the presence of a grand, five orders of magnitude growth of human population (from \( 10^5 \) to \( 10^{10} \)), only just a several times change of the mentioned parameters results in a little relative deviation from the ideal case regarding \( m \) and \( w \) as constant. Admissibility of this idealization is conditioned by that the species abundances in preceding epochs are known only with the accuracy within one order of magnitude so that several times variations (but less than one order of magnitude) due to the drifting of parameters \( m \) and \( w \) lie within the range of uncertainty.

As far as the humanity is concerned, hyperbolic growth of population is provided with domination of neural memory and participation of neural processor in producing knowledge.

According to estimations of Foerster, Mora, and Amiot (1960), the moment of singularity \( t_i \) falls on the end of 2026 (i.e. approximately \( t_i = 2027 \)) and the quantity reciprocal to the specific rate of knowledge production, found from empirical data, is equal to \( w^{-1} = 200 \) billion people×years. Calculations using updated demographic data yield a corrected value of \( w^{-1} = 215 \) billion people×years (Markov and Korotaev, 2009).

It should be taken into account that the hyperbolic growth of humanity characterizes a long-term trend of its population size and does not describe short-term processes (such as cycles or fluctuations) yielding deviations from the average value. This is in accordance with Malthus’ statement that “population is necessarily limited by means of sub-
sistence” (Malthus, 1826, item I.II.22). Malkov, Korotaev, and Khalturina (2007) reformulated this statement as follows: “the growth of human population at a given moment of time is restricted by the top of Earth’s carrying capacity being determined by the currently observed development level of vital technologies”. The top of carrying capacity $N(t)$ just describes the long-term trend, following a slow increase in the technological development level, where $t$ is the large-scale (or historical) time. At a given technological level, the current human population $n(\tau)$ (where $\tau$ is the small-scale, or local, time) can suffer significant variations much faster than the rate of technological development. This can be illustrated by the Malthusian cycles as “excess of resource – fast growth of population – exhaustion of resource – fast fall in population”, or by such factors as epidemics and natural catastrophes. The scale of local time $\tau$ is essentially smaller than that of historical time $t$ (Fig. 3). Local processes can be associated, for example, with a deviation from the top of carrying capacity and relaxation into equilibrium. Such processes are described by the Verhulst equation $dn/d\tau = rn[1 - n/N(t)]$. It is the growth of $N(t)$ in the historical time-scale that reveals the informational essence of civilization.

![Fig. 3. Schematic of demographic processes of different scales.](image)

5. HYPERBOLIC GROWTH OF BIODIVERSITY

Biodiversity of a community is adopted to estimate by Shannon’s entropy

$$H = -\sum_{i=1}^{N} (n_i / n) \log_2 (n_i / n),$$

(1)
where \( N \) is the number of taxa, \( n_i \) is the \( i \)th taxon abundance, \( n \) is the total community abundance, \( n = n_1 + \ldots + n_N \). Entropy achieves its maximum value \( H = \log_2 N \) at equal abundances of taxa \( n_i = \text{const} \). It is seen that the maximum entropy depends only on the number of taxa. The stated interpretation of biodiversity suffers from the shortcoming that equilibrium abundances of taxa cannot in reality be equal because of essential differences in taxa’ properties, primarily memory sizes.

During a major part of the evolution of biosphere, genetic memory predominated. Memory size is important for accumulation of valuable (vital) information necessary for survival. However, genome size is not always directly connected to the size of valuable information because a significant part of genome is occupied by a weakly structured information called the “junk DNA”. Valuable information occupies the rest part of genome which is commonly called the “non-redundant genome” (Adami et al., 2000; Sharov, 2006). The size of non-redundant genome is a measure of biological complexity of organism. Markov and Korotaev (2009) proposed to use the minimum size of genome in a group of species belonging to the same taxon as an approximation to the poor determined size of non-redundant genome.

The junk part of genome is most likely of small importance for a homeostatic community because for supporting equilibrium in a weakly fluctuated environment it is sufficient to use only valuable information accumulated in genome. However, the storage of raw information, maybe useless in some period, can appear to be needed in the future, at a significant change of ambient conditions. Support of a large-size genome requires increasing cell sizes because of additional energetic expenditures for DNA replication as well as for maintaining and reproducing a lot of extra proteins. On the other hand, storage of raw information is useful in the evolutionary process because junk DNA contains mobile genetic elements (MGEs), facilitating genetic variability, maybe useful for organism. Therefore, considering the long-term knowledge production it is necessary to account for the accumulation of partially processed information in genome just realized in the form of MGEs. Without this accumulation, knowledge production will go much slower, weakening the competitiveness of taxon. Thus, a definite compromise should be kept between the size of non-redundant part of genome and the size of accumulated MGEs.

Individuals composing a taxon have close genetic characteristics including the genome size. Let \( \mu_i \) is the size of non-redundant genome in the \( i \)th taxon. Main genetic memory of the taxon (involving only non-redundant part of genome) equals \( m_i = \mu_i n_i \) and
the total memory of all $N$ taxa in the community is the sum $M = m_1 + ... + m_N$. Shannon’s entropy accounting for different states of memory can be expressed as

$$H = -\sum_{i=1}^{N} (m_i / M) \log_2 (m_i / M). \quad (2)$$

It should be noted that formula (2) resembles the exergy introduced by Jørgensen (1995) and accounting for the information stored in the ecosystem structure.

In equilibrium, entropy (2) achieves its maximum $H = \log_2 N$ (the same as for entropy (1)); memory sizes of taxa being equal $\mu_i n_i = m = \text{const}$ and total memory of community becoming proportional to the number of taxa in it: $M = mN$. Thus, in the state of homeostasis the number $N$ of viable taxa again becomes the crucial parameter of biodiversity, however now this state is supported not by equal abundances, but by equal memory sizes of coexisting taxa. This conclusion is important for the biosphere because it usually takes a nearly homeostatic state, interrupted for a short while with rare catastrophic events.

The same conclusions can be deduced from the following qualitative consideration. Stability of a taxon correlates with its lifetime, which increases with accumulating valuable information in memory. Therefore, memory size becomes an important criterion of selection in the evolutionary process. Taxa having less memory sizes than other members of the community lose competitiveness and go to extinction. As a result of this selection, memory sizes of coexisting taxa are equalized with time. However, this is not a strict equalizing by values but only an equalizing by orders of magnitudes. Strict equalizing is generally impossible because ambient conditions suffer various disturbances: from short-term fluctuations to long-term waves. Equalizing of memory sizes can be provided both by a slow change of genome size and also by a much faster way – by allowing the taxa to compensate underdevelopment of genomes by means of increasing their abundances. In this case, the total memory size of community $R$ will be proportional to the number of taxa $N$, i.e. $R \sim N$. As mentioned above, the non-redundant part of genome is the most important in supporting equilibrium, therefore in this case the memory of community $R$ should be considered only as the non-redundant memory $M$.

Thus, taxa tend to support their memory sizes on the average level of community. However, from the viewpoint of survival, this is only a necessary condition, not a sufficient one. Permanent variations of ambient conditions require from the organisms to elaborate adaptive reactions and hence to permanently produce knowledge with the rate not less than the average level of rates of taxa in the community, otherwise competition will be lost.
Therefore, in the evolutionary process there will be equalized (by orders of magnitude) not only memory sizes but also rates of knowledge production in all taxa. For the rate of knowledge production it is important to take into account both factors: the amount of valuable information accumulated in genome and the storage of MGEs accelerating the process. As a result of the equalizing of rates of different taxa, the total knowledge production of community will be equal to \( dq / dt = wN \), where \( w \) is the average knowledge production rate per taxon. In respect to the law of logarithmic compression of information, \( q = \ln R \) (compression is carried out in the coding of valuable information in genome), we can come to the same laws concerning the rate of knowledge accumulation \( dq / dt \sim e^q \), growth rate \( dN / dt = wN^2 \) and hyperbolic time-dependence of the number of taxa \( N = w^{-1}(t_i - t)^{-1} \) like for humanity, but with different values of parameters. If we specify taxa as genera, values of parameters for the Phanerozoic biota will be: \( w^{-1} = 4.34 \cdot 10^5 \) taxa\(^{-1}\) million years, \( t_i = 30 \) million years (Markov and Korotaev, 2009).

Thus, in the evolutionary process there are two factors of selecting subjects of civilization: memory size \( m \) and knowledge production rate \( w \), which have to be equalized in the community, thereby providing the expansion of the total memory size of civilization \( R \) and the total knowledge production rate \( dq / dt \) according to the hyperbolic law.

6. TRANSITIONS WITH CHANGING OF DOMINANT MEMORY TYPE

The hyperbolic growth of biodiversity had continued until there appeared a taxon in the biosphere who could develop neural memory (mainly, due to the development of social relations – Herrmann et al., 2009), became using it actively for producing and accumulating knowledge, and, due to that, acquired monopolistic positions in the biosphere (we mean, of course, the human).

Transition from the domination of genetic memory in the biota to the domination of neural memory in one of the taxa led to a qualitative change in the evolutionary process: to the beginning of hyperbolic growth of dominant taxon’s abundance and to the cessation of hyperbolic growth of biodiversity (because the dominant taxon monopolizes and destroys biosphere).

At present, we can observe one more transition connected with the development of external memory. This type of memory has suffered several qualitative changes. Initially, it
was used on early stages of anthropogenesis – we mean subjects of labor as samples for reproduction. External memory has got a noticeable development with the appearance of writing and especially with the beginning of book-printing era. Now, the amount of accumulated printed matter is $\sim 10^{14}$ bits. However, the most powerful growth of external memory has occurred only in recent times. This process has been initiated by the creation of computers and continuing enhancement of their facilities (memory and speed). Till the end of XX century, total external memory achieved the level of neural memory of humanity and then surpassed it (Fig. 2). Development of this type of memory will lead to the consequences similar to that were in the previous transition, namely to the appearance of a dominant taxon, already not in the biota but in the human world community. Apparently, this will be a country (rather even a group of countries) in which knowledge production develops most intensively. However, it results not in growth of population of this country (since accumulation of knowledge essentially increases requirements to the quality of life, sharply raises its cost and finally causes a reduction of fertility), but in growth of its informational and, naturally, economic power. In this process, the size of external memory $R$ will increase exponentially with growth of knowledge: $R \sim e^q$ (it is a consequence of the above-mentioned logarithmic compression of information in production of knowledge) and knowledge production rate will grow proportionally to memory size, i.e. $dq/dt = wR \sim e^q$, where $w$ is the specific knowledge production rate (per unit of external memory size). This yields the equation of memory growth $dR/dt = wR^2$, which is followed by the hyperbolic time-dependence of memory size: $R = w^{-1} (t_1 - t)^{-1}$, where $t_1 = (wR_0)^{-1}$ is the moment of singularity, and $R_0$ is the initial size of external memory (at $t = 0$).

To estimate parameters, we take the year 2002 as the initial one, because for it we have data on knowledge production rate $v = dR/dt$. From the relationships $v = wR_0^2$ and $t_1 = (wR_0)^{-1}$, we obtain $w = v/R_0^2$ and $t_1 = R_0/v$. According to the above estimates $v \sim 10^{20}$ bits/year, $R_0 \sim 10^{21} - 10^{22}$ bits; these values characterize the external memory based mainly on magnetic carriers. Using these values, we obtain $w^{-1} \sim 10^{22} - 10^{24}$ bits×years, $t_1 \sim 10^4 - 10^7$ years. Thus, in the development of external memory on magnetic carriers, moment of singularity will be achieved in several tens of years. Specification of the carrier type (magnetic) is important, first, because it is the type that is now the basic one and, second, because the appearance of more advanced carriers on an essentially distinct basis will lead to a change of knowledge production rate.
A summary of parameter values for different types of memory presents in Table. It should be noted that the specific rate $w$ of knowledge production by biota (per taxon) and by humanity (per person) have the same order of magnitude, only just two times differing – the human produces knowledge faster (emphasize that Table presents reciprocal values: $1/w$).

Thus, the appearance and development of every new type of memory leads to cardinal changes in the structure of old civilization and to the rise from its depths of a new civilization with domination of this new type of memory.

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